

# Scintillation Effects in High Energy Laser Propagation: A Numerical Approach

Paulo Jorge Duda de Moraes  
University of São Paulo  
São Paulo, Brazil  
orcid.org/0000-0002-4336-2963

Rubens Cavalcante da Silva  
University of São Paulo  
São Paulo, Brazil  
orcid.org/0000-0002-9794-0992

Andre Carvalho  
University of São Paulo  
São Paulo, Brazil  
orcid.org/0000-0001-8569-9532

Wagner de Rossi  
IPEN-CNEN/SP  
São Paulo, Brazil  
orcid.org/0000-0003-1371-7521

Claudio Costa Motta  
University of São Paulo  
São Paulo, Brazil  
orcid.org/0000-0002-2508-7320

**Abstract**—A modeling of a laser beam propagation through the atmosphere has been developed and is reported in this paper. The effects of scintillation are numerically simulated in a coupled numerical solution. This solution has been done by using Navier-Stokes, paraxial equations and a phase-screen model.

**Index Terms**—scintillation, thermal blooming, refractive index, paraxial equation, Navier-Stokes equations

## I. INTRODUCTION

Nowadays, high energy lasers (HEL) have many applications, for example: satellite communication [1], wireless power transmission [2] and remote recharge of batteries [3]. The fluid-laser interaction modeling for these uses is essential to predict the laser beam quality and the power density at the receiver. So, to improve and to predict results for some of these applications an investigation of phenomena related to the atmospheric propagation of the laser beam is required [1]. Some recent researches have been started to develop numerical solutions whose velocity field and temperature fluctuations are treated as dynamic variables [4], [5], in contrast to previous studies where these variables were prescribed [6], [7].

Scintillation is random fluctuation in the atmosphere refractive index. These scintillations are directly associated with temperature fluctuations caused by the creation of eddies in the atmosphere as a consequence of the random distribution of increased temperature fluctuations.

This work adds to the model developed in [4] the phenomenon of scintillation using a phase screen scheme. The distribution of temperature variations introduced in the model is based on the Kolmogorov's theory. Furthermore, the Navier-Stokes equations (N-S) are solved taking into account the interaction between the mean flow and these temperature distributions.

## II. FORMULATION

The basic model considers the paraxial wave equation and the N-S Equations to calculate the variables of interest. The paraxial wave equation for the electric field is described as follows:

$$\left(2jk\eta_s \frac{\partial}{\partial z} + \nabla_{\perp}^2 + 2\eta_s \eta_f k^2\right) V = 0 \quad (1)$$

where,  $n_s$  is the mean refractive index,  $n_f$  is a small correction to the refractive index,  $k$  is the wavenumber of the beam and  $\nabla_{\perp}^2$  is the Laplacian in the transverse  $xy$ -plane.

The eq. (1) is coupled with the N-S:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (2)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{\partial P}{\partial x} + \frac{1}{Re} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \quad (3)$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{\partial P}{\partial y} + \frac{1}{Re} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + RiTe_2 \quad (4)$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{1}{RePr} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + St|V|^2 \quad (5)$$

where  $u$ ,  $v$ ,  $P$  and  $T$  are the velocity in direction  $x$ , the velocity in direction  $y$ , scalar pressure and temperature fields, respectively and the dimensionless numbers are: Reynolds (Re), Peclet (Pe), Richardson (Ri) and Stanton (St).

To incorporate scintillation effects in the solver the variables are decomposed in a power series in first order approximation:

$$u(x, y, t, \delta) = u_0(x, y, t) + \delta u_1(x, y, t) \quad (6)$$

$$v(x, y, t, \delta) = v_0(x, y, t) + \delta v_1(x, y, t) \quad (7)$$

$$T(x, y, t, \delta) = T_0(x, y, t) + \delta T_1(x, y, t) \quad (8)$$

$$P(x, y, t, \delta) = P_0(x, y, t) + \delta P_1(x, y, t) \quad (9)$$

where  $\delta$  is a scintillation parameter.

Hence, to obtain the final formulation of the model, the equations (6) to (9) are substituted into eq. (1) to (5).

To incorporate the turbulence into  $T_1$ , the temperature fluctuations term, in eq. (8), it is necessary to generate power spectral density (PSD) based in Kolmogorov theory by using a random function to each  $k_x$  and  $k_y$  in eq. (10), [1]:

$$T_1(x, y) = \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} (r(k_{xm}, k_{ym}) + ji(k_{xm}, k_{ym})) \times \sqrt{\frac{0,49r_0^{-5/3} e^{-\frac{k_x^2 + k_y^2}{k_m^2}}}{L_x L_y (k_{xm}^2 + k_{ym}^2 + k_0^2)^{11/6}}} e^{i(k_{xm}x + k_{ym}y)} \quad (10)$$

where  $r(k_{xm}, k_{ym})$  and  $i(k_{xm}, k_{ym})$  are standards gaussian distributions,  $k_{xm}$  and  $k_{ym}$  are the discretized scalar spatial wave numbers in transversal directions,  $k_m = \frac{5.93}{l_0}$ ,  $k_0 = \frac{2\pi}{L_0}$ ,  $r_0$  is the atmospheric coherence diameter,  $l_0$  is the inner scale of turbulence and  $L_0$  is the outer scale of turbulence.

In order to incorporate scintillation into the electric field, a phase screen scheme is introduced at each step in the axial direction as follows:

$$V(z + \Delta z) = V(z) e^{i\delta T_1(z)} \quad (11)$$

where  $V$  is defined in eq. (1).

### III. NUMERICAL RESULTS

First, the set of dimensionless numbers: Reynolds,  $Re = 1000$ ; Prandtl,  $Pr = 1$ ; Richardson,  $Ri = 6 \times 10^5$ ; and Stanton,  $St = \frac{1}{3}$  is used in model presented in [4]. The initial laser beam distribution is defined by:

$$V(x, y, z = 0, t = 0) = e^{-\frac{\pi}{2}(x^2 + y^2)} \quad (12)$$

The coupled N-S and the paraxial equation are solved by means of the finite difference method with the split-step Fourier method [4]. The parameters to obtain the numerical solution are described in [4].

Equation (10) produces the coefficients for  $T(\omega_x, \omega_y)$ , where  $\omega_x$  and  $\omega_y$  are angular frequencies, which can be seen in the Fig. 1a. According to [8], the turbulence is described by the following parameters: outer scale is  $L_0 = 1$  [m] and the inner scale  $l_0 = 1$  [cm] in eq. (10), [8].

Afterwards, by applying the FFT in the data shown in Fig. 1, the temperature fluctuations in transversal direction are obtained (Fig. 1b).

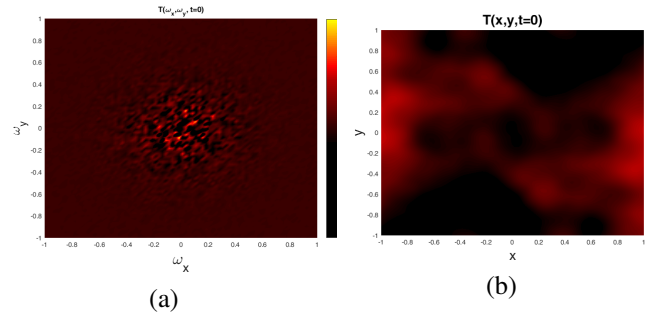


Fig. 1. Temperature fluctuations  $T_1(z = 0, t = 0)$  with  $Re = 1000$ ,  $Pe = 1000$ ,  $Ri = 10.e4$  with  $St = \frac{1}{30}$ ,  $L_0 = 1$  [m] and  $l_0 = 1$  [cm] (a)  $T_1(\omega_x, \omega_y)$  based in eq. (10) (b)  $T_1(x, y)$  initial fluctuations for temperature in transverse plane.

Applying the data shown in Fig. 1 in eq. (11), with the initial electric field amplitude given by Eq. (12), the effect of scintillation over the laser beam intensity is obtained (Fig. 2)

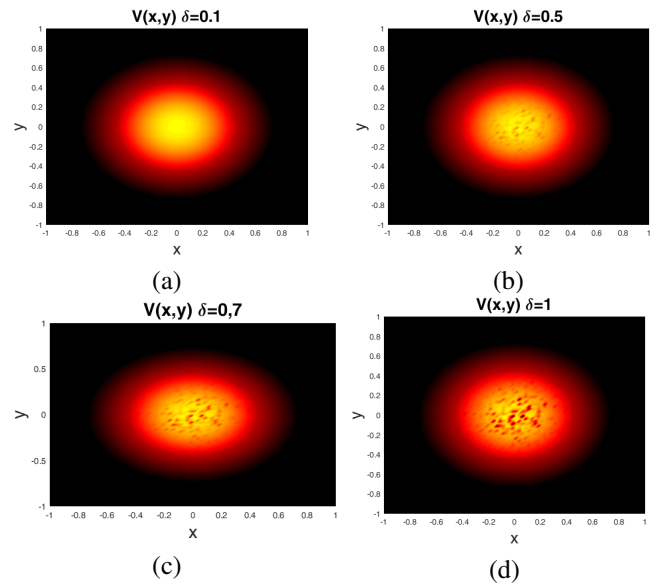


Fig. 2. Laser beam intensity  $V(x, y, z = 1, t = 5 \times 10^{-5})$  using coupled laser-fluid solver presented in [4] plus scintillation effects varying parameter  $\delta$ . (a)  $V(x, y, z = 1, t = 5 \times 10^{-5})$  for  $\delta = 0.1$ . (b)  $V(x, y, z = 1, t = 5 \times 10^{-5})$  for  $\delta = 0.5$ . (c)  $V(x, y, z = 1, t = 5 \times 10^{-5})$  for  $\delta = 0.7$ . (d)  $V(x, y, z = 1, t = 5 \times 10^{-5})$  for  $\delta = 1$

Fig. 2 shows the initial condition of the model, where the laser beam distribution is under no effect of scintillation. In such condition, the dimensionless grid screen and time-step are, respectively,  $\Delta z = 1$  and  $\Delta t = 5 \times 10^{-5}$ . The unique parameter which was changed from Fig. 2a to 2c is the scintillation factor  $\delta$ . As  $\delta$  increases from 0.1 to 1, the scintillation effect which is almost imperceptible in Fig. 2a started to be relevant in Fig. 2c.

In order to analyze the effects of scintillation across the time evolution of the solver, which was explained in [4], a study with the same set of dimensionless numbers and a fixed scintillation factor  $\delta = 0.1$  was developed. As shown in Fig. 3, the inclusion of scintillation in the time evolution promotes in the solution, as time evolves, a greater relevance of random

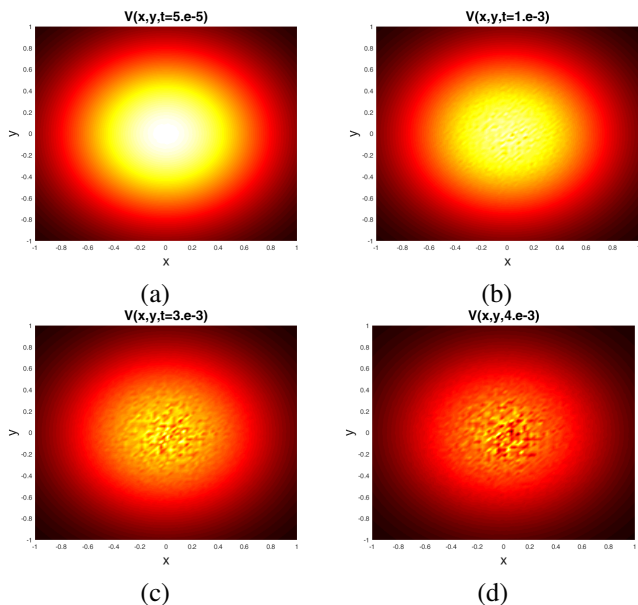


Fig. 3. Time evolution of the electric field amplitude  $V(x, y)$  coupled solver including the effects of scintillation for a factor scintillation  $\delta = 0.1$ . a)  $V(x, y, z = 1, t = 5.e - 5)$ , b)  $V(x, y, z = 20, t = 1.e - 3)$ , c)  $V(x, y, z = 60, t = 3.e - 3)$ , d)  $V(x, y, z = 80, t = 4.e - 3)$

temperature variations for this  $\delta$  when it is compared to the thermal blooming of the solver described in [4].

The reason for this observation is the magnitude of scintillation factor, which is used  $\delta = 0.1$ . The objective in presenting the Fig. 3 was to highlight the random phase changes in the model. To observe the coupling of thermal blooming with scintillation two factors are needed in this solver: a reduction in magnitude of the scintillation factor and a longer time evolution than Fig. 3d,  $t = 4.e - 3$ , which will be done in future works.

## CONCLUSION

A numerical analysis of a gaussian laser beam atmospheric propagation was carried out and the effects of scintillation in the laser propagation was described. An analysis of the coupling of thermal blooming and scintillation effects was shown.

## REFERENCES

- [1] L. C. ANDREWS and R. L. Phillips, *Laser beam propagation through random media*, second edition ed. Bellingham: SPIE-International Society for Optical Engineering, 2005.
- [2] T. He, G. Zheng, Q. Wu, H. Huang, L. Wan, K. Xu, T. Shi, and Z. Lv, "Analysis and experiment of laser energy distribution of laser wireless power transmission based on a powersphere receiver," in *Photonics*, vol. 10, no. 7. MDPI, 2023, p. 844.
- [3] T. Ojha, T. P. Raptis, A. Passarella, and M. Conti, "Wireless power transfer with unmanned aerial vehicles: State of the art and open challenges," *Pervasive and Mobile Computing*, p. 101820, 2023.
- [4] P. J. D. De Morais, R. C. Da Silva, W. De Rossi, and C. C. Motta, "Numerical solution of atmospheric laser beam propagation using artificial compressibility and pseudo-spectral methods," in *2022 SBFTON IOPC*. IEEE, 10 2022, pp. 1–5. [Online]. Available: <https://ieeexplore.ieee.org/document/9992726/>

- [5] C. D. Morrill and B. Akers, "High Energy Laser Propagation: Modeling Scintillation Effects," in *Imaging and Applied Optics 2019 (COSI, IS, MATH, pcAOP)*, vol. Part F164-. Washington, D.C.: OSA, 2019, p. PTh2D.3. [Online]. Available: <https://opg.optica.org/abstract.cfm?URI=pcAOP-2019-PTh2D.3>
- [6] D. Smith, "High-power laser propagation: Thermal blooming," *Proceedings of the IEEE*, vol. 65, no. 12, pp. 1679–1714, 1977.
- [7] J. Gustafsson, B. F. Akers, J. A. Reeger, and S. S. Sritharan, "Atmospheric propagation of high energy lasers," *Engineering Mathematics Letters*, 2019.
- [8] C. Coulman, J. Vernin, Y. Coqueugniot, and J. Caccia, "Outer scale of turbulence appropriate to modeling refractive-index structure profiles," *Applied optics*, vol. 27, no. 1, pp. 155–160, 1988.